# 3-dimensional Eigenmodal Analysis of Electromagnetic Structures


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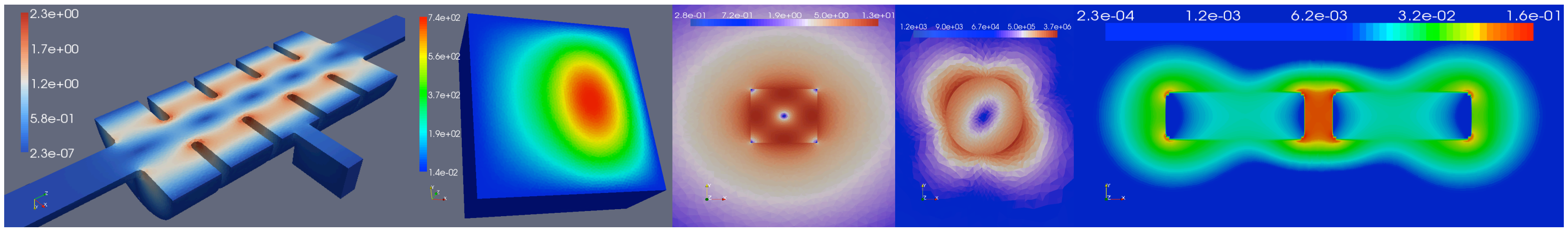


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# Outline

- Introduction of Femaxx
- Resonant cavities
  - Jacobi-Davidson for linear eigenvalue problems (JDQZ)
- Microwave antennas
  - Jacobi-Davidson for quadratic /nonlinear eigenvalue problems(NLJD)
- Summary

# Introduction of Femaxx

- A large scale software project.
- Developed at ETH Zurich and PSI.
- Simulation tool for electromagnetic structures.
- Parallelized for distributed memory computers.
- Solves electric field vector wave equation.
- Finite element method (FEM).
- 3-dimensional, unstructured tetrahedral mesh.
- Model arbitrary geometry or material property.
- Linear, quadratic and nonlinear eigensolvers.
- Open source software, http://amas.web.psi.ch/docs/femaxx-doc/html/ .

Swiss Federal Institute of Technology (ETH Zurich)

Paul Scherrer Institute (PSI)

# Lossless Resonant Cavities

- Time-harmonic electric field *curl-curl* equation

$$\nabla \times \left( \mu_r^{-1} \nabla \times \boldsymbol{E}(\boldsymbol{x}) \right) - k_0^2 \varepsilon_r \boldsymbol{E}(\boldsymbol{x}) = \boldsymbol{0}, \quad \boldsymbol{x} \in \Omega.$$

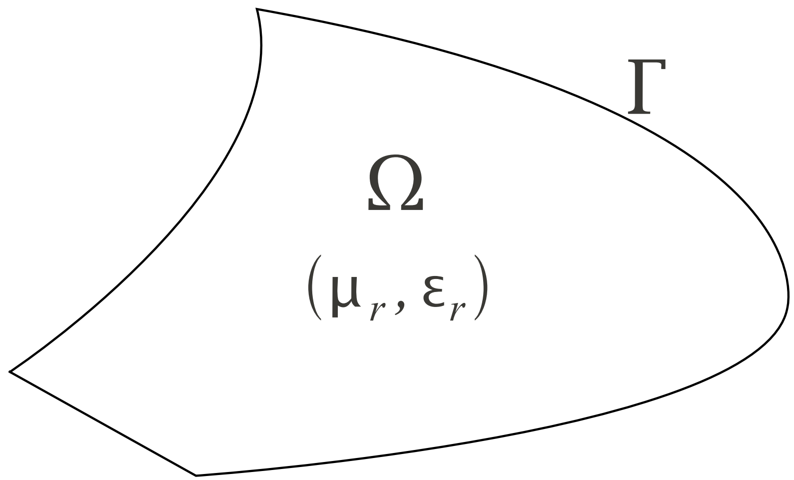
- Divergence free condition

$$\nabla \cdot \left( \varepsilon_r \boldsymbol{E}(\boldsymbol{x}) \right) = \boldsymbol{0}, \quad \boldsymbol{x} \in \Omega.$$

- Perfect electric conductor (PEC) boundary condition

$$\boldsymbol{n} \times \boldsymbol{E}(\boldsymbol{x}) = \boldsymbol{0}, \quad \boldsymbol{x} \in \Gamma.$$

$k_0$ : wavenumber in free space
$\mu_r$ : magnetic relative permeability
$\varepsilon_r$ : electric relative permittivity
$\Omega$ : computational domain
$\Gamma$ : boundary conductor wall



# Lossless Resonant Cavities

- FEM yields a *real-valued linear* eigenvalue problem

$$A\,\boldsymbol{x} = \lambda M\,\boldsymbol{x}, \quad C^T \boldsymbol{x} = \mathbf{0}, \quad \lambda = k_0^2.$$

$A$ is positive semi-definite: $a_{ij} = \int_\Omega \mu_r^{-1} (\nabla \times \boldsymbol{N}_i) \cdot (\nabla \times \boldsymbol{N}_j)\, d\boldsymbol{x}.$

$M$ is positive definite: $m_{ij} = \int_\Omega \varepsilon_r \boldsymbol{N}_i \cdot \boldsymbol{N}_j\, d\boldsymbol{x}.$

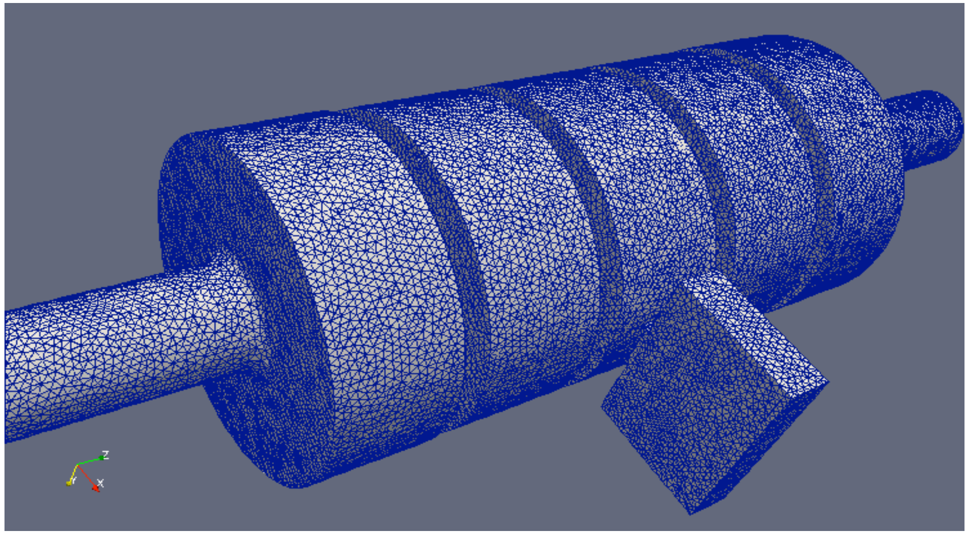
$C$ is a rectangular matrix: $c_{il} = \int_\Omega \varepsilon_r \boldsymbol{N}_i \cdot \nabla N_l\, d\boldsymbol{x},\ C = MY.$

$\boldsymbol{N_i}$ is Nédélec basis function, $N_l$ is Lagrange basis function,

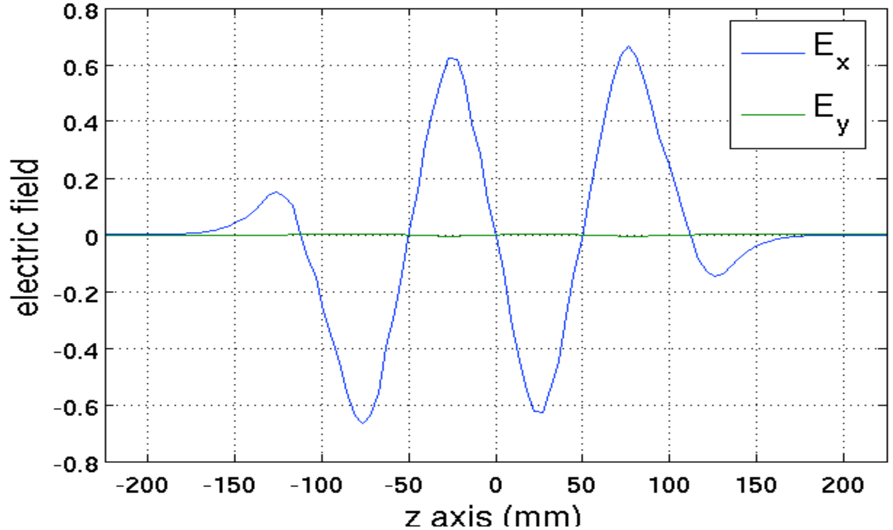
$Y$ is a rectangular matrix of $y_{jl}$: $\nabla N_l = \sum_{j=1}^{m} y_{jl} \boldsymbol{N}_j.$

- JDSYM eigensolver: Jacobi-Davidson method for real-valued symmetric eigenvalue problem.

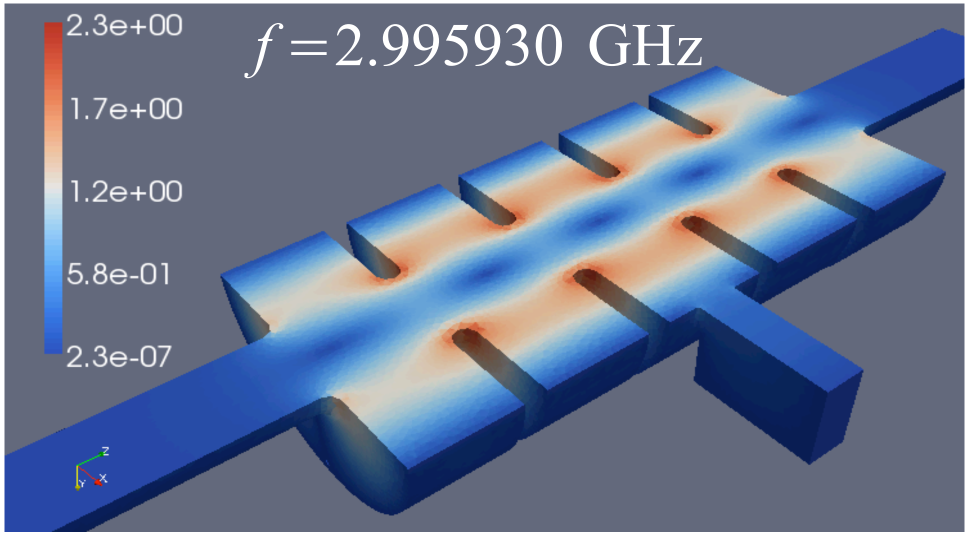
# 5-cells Transverse Deflecting Cavity



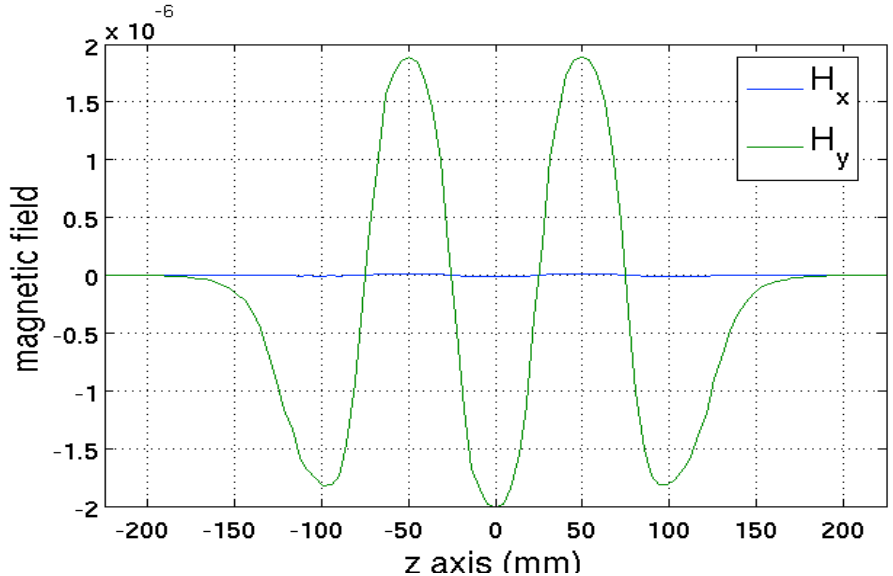
(a) mesh containing 872'261 elements.




(c) $E_x$ and $E_y$ over z-axis.



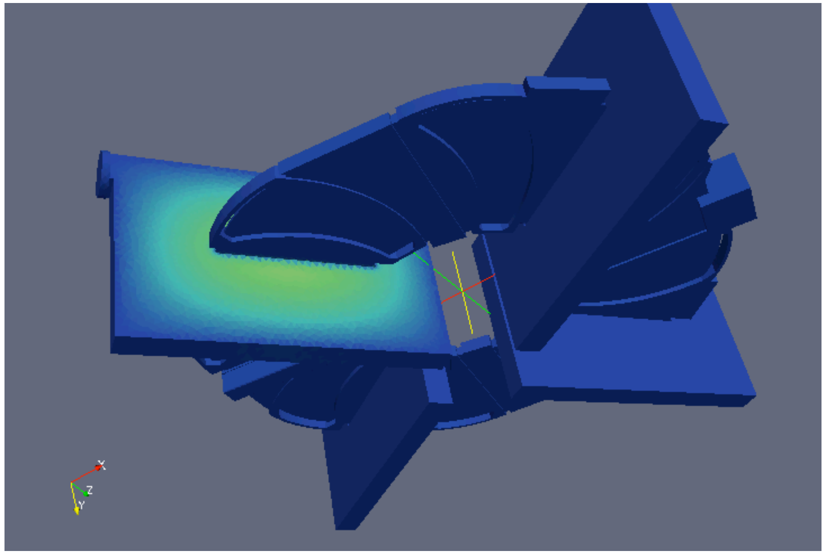

(b) electric field distribution of TM110 mode.



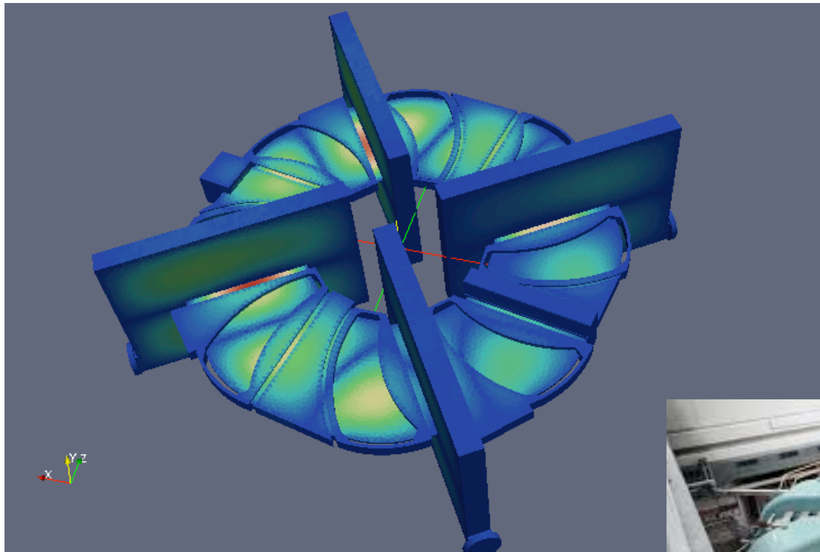

(d) $H_x$ and $H_y$ over z-axis.

# Cyclotron at PSI

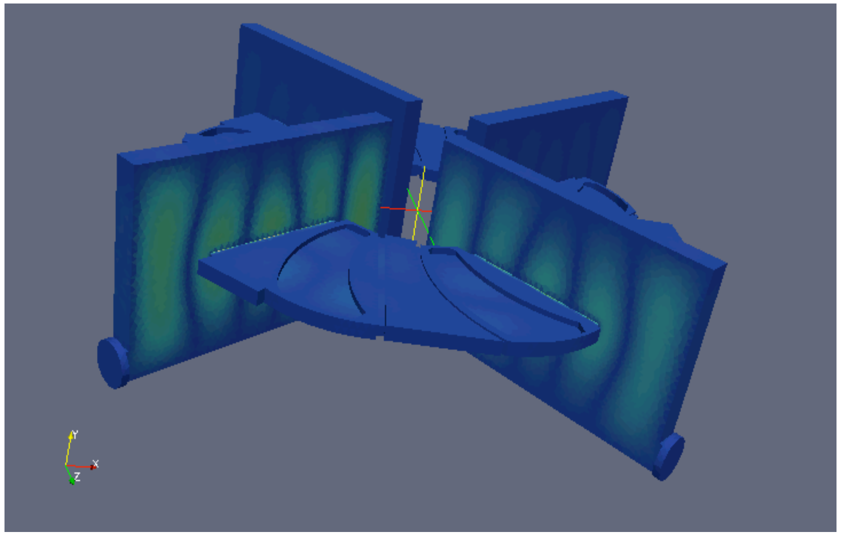
Tetrahedral mesh created from original geometry obtained as STEP file from CAD.



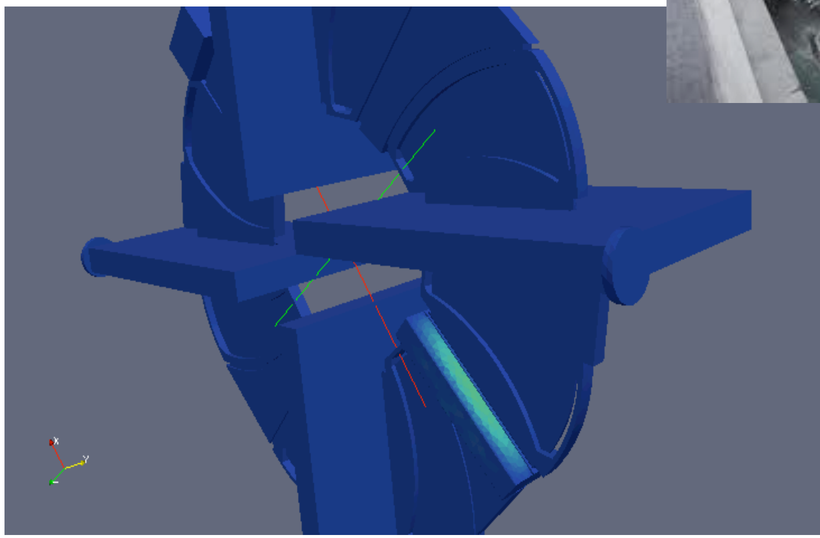
(a) dominant mode: 51.48 MHz



(b) 31st mode: 101.04 MHz

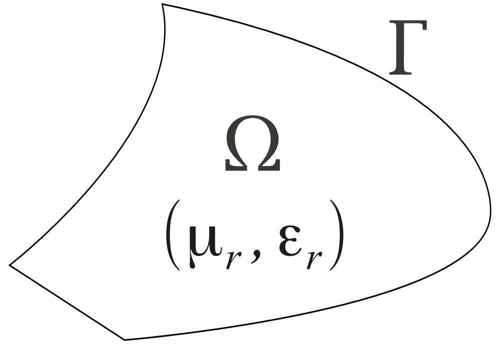


(c) 86th mode: 149.58 MHz



(d) 87th mode: 151.51 MHz

# Dielectric Lossy Material




- Complex-valued relative permittivity

$$\varepsilon_r = \mathrm{Re}(\varepsilon_r) - i\,\mathrm{Im}(\varepsilon_r).$$

- Perfect electric conductor boundary condition

$$\boldsymbol{n} \times \boldsymbol{E}(\boldsymbol{x}) = \mathbf{0}, \quad \boldsymbol{x} \in \Gamma.$$

- FEM yields constrained eigenvalue problem

$$A\,\boldsymbol{x} = \lambda M\,\boldsymbol{x},$$
$$C^T \boldsymbol{x} = \mathbf{0}.$$

- Entries of M and C depend on $\varepsilon_r$

$$m_{ij} = \int_\Omega \varepsilon_r \boldsymbol{N}_i \cdot \boldsymbol{N}_j \, d\boldsymbol{x}, \qquad c_{il} = \int_\Omega \varepsilon_r \boldsymbol{N}_i \cdot \nabla N_l \, d\boldsymbol{x}.$$

A is *real* symmetric; M is *complex* symmetric;
C is a *complex* rectangular matrix.

# Jacobi-Davidson QZ Eigensolver

JDQZ solves generalized *non-Hermitian* eigenvalue problem.

```
choose start vector x_0;   it = 0.
when it ≤ it_max do
    Solve projected eigenproblem with QZ decomposition.
    Compute an eigenpair closest to target: τ.
    if the associated residual is small enough
        Accept this eigenpair; deflate search space.
    end if
    Restart if necessary.
    Find an approximate solution t of correction equation.
    Project t in null space of C^T.
    Orthonormalize t against previous search vectors.
    Expand search space with t.  it++.
end
```

Krylov solver

Divergence-free condition

Modified Gram-Schmidt

Ref: Z. Bai, J. Demmel, J. Dongarra, A. Ruhe and H. van der Vorst. *Templates for the Solution of Algebraic Eigenvalue Problems*. SIAM, Philadelphia, PA, 2000.

# Implementation of Femaxx

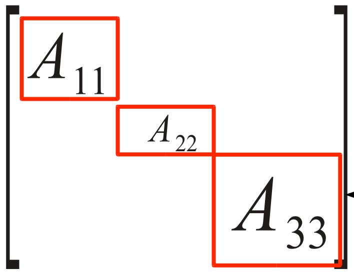
- C++ : object-oriented design pattern
- Gmsh : mesh generator
- ParMETIS : mesh partitioner
- Paraview : visualization of electromagnetic field
- Trilinos : parallel objects

# Trilinos in Femaxx

- Epetra
  - parallel multivectors
  - parallel sparse matrices
  - distribution map
  - uses MPI functionality
- AztecOO
  - Iterative solvers for correction equation
- Ifpack
  - Block Jacobi preconditioners

# Ifpack

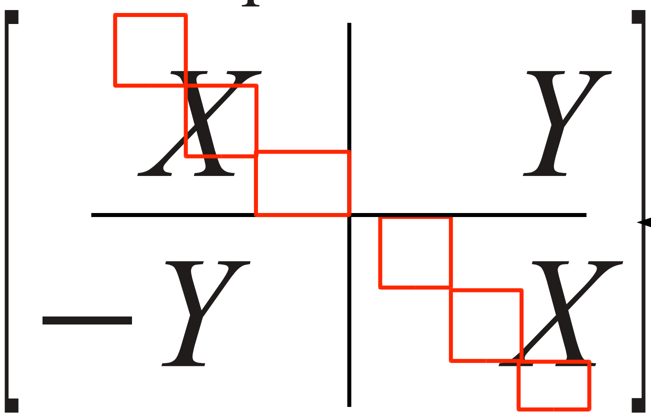
- Block Jacobi preconditioner
  - Extract diagonal blocks by LU and ILU.




solve each block by LU or ILU

  - Ifpack doesn't support Complex arithmetic.

$$\text{Complex matrix}, \quad Z = X + iY, \quad \text{double-sized} = \begin{bmatrix} X & Y \\ -Y & X \end{bmatrix}.$$




Ifpack extracts only real part of $C$ .

# Construct double-sized real valued matrix

- Alternative approach :

$$Z = X + iY, \quad z_{ij} = x_{ij} + iy_{ij}, \qquad Z = \begin{bmatrix} z_{11} & 0 & z_{13} \\ 0 & z_{22} & z_{23} \\ z_{31} & 0 & z_{33} \end{bmatrix}.$$

$$\text{double-sized} = \begin{bmatrix} x_{11} & y_{11} & 0 & 0 & x_{13} & y_{13} \\ -y_{11} & x_{11} & 0 & 0 & -y_{13} & x_{13} \\ 0 & 0 & x_{22} & y_{22} & x_{23} & y_{23} \\ 0 & 0 & -y_{22} & x_{22} & -y_{23} & x_{23} \\ x_{31} & y_{31} & 0 & 0 & x_{33} & y_{33} \\ -y_{31} & x_{31} & 0 & 0 & -y_{33} & x_{33} \end{bmatrix}.$$

  - Ifpack can extract block diagonal elements correctly.

Reduction of 5% iterations in JDQZ



Ref: David Day, Michael A. Heroux, *Solving Complex-Valued Linear Systems via Equivalent Real Formulations*, SIAM J. Sci. Comput., Vol.23, No.2, pp.480-498.

# Ifpack

- Construct proconditioners for Krylov solver
  - Block Jacobi AdditiveSchwarz with LU factorization
    - overlap = 0;
    - partitioner : linear;
    - LU solver : Amesos_Klu;
  - Block Jacobi AdditiveSchwarz with ILU factorization
    - overlap = 0;
    - partitioner : linear
    - ILU solver : Ifpack_ILU

Ref: Marzio Sala, Michael Heroux, *Robust Algebraic Preconditioners using IFPACK 3.0*, Sandia Report, SAND2005-0662, 2005.

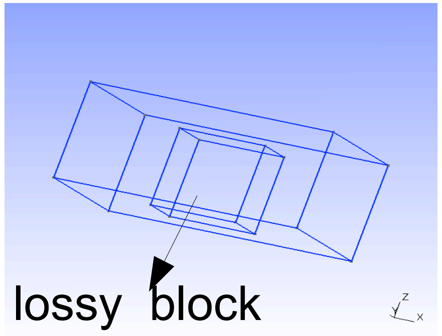
# Example of dielectric lossy material

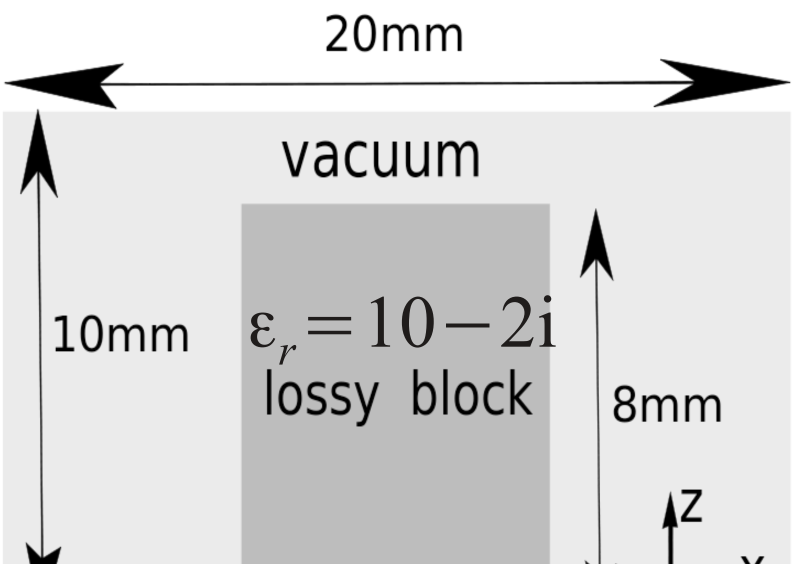
- **#Tetrahedra** : 141'149
- **Processors** : 32
- **Quadratic element order**
- **Number of eigenvalues** : 6
- **Tolerance** : 1.0e-6
- **Target** : [1.0]
- **Krylov solvers** : Bi-CGStab
- **Preconditioners** : Diagonal, Block Jacobi-LU/ILU
- **Max iteration of correction equation** : 50



- **Jmin : 5, Jmax** : 10

# Lossy Dielectric Block




Stored energy: $U = \frac{\varepsilon_0}{2} \int_\Omega \mathrm{Re}(\varepsilon_r) |\boldsymbol{E}|^2 \, d\boldsymbol{x}$.

Dissipated energy: $U_d = \frac{\varepsilon_0}{2} \int_\Omega \mathrm{Im}(\varepsilon_r) |\boldsymbol{E}|^2 \, d\boldsymbol{x}$.

Resonant frequency: $f = \frac{\omega}{2\pi}$.

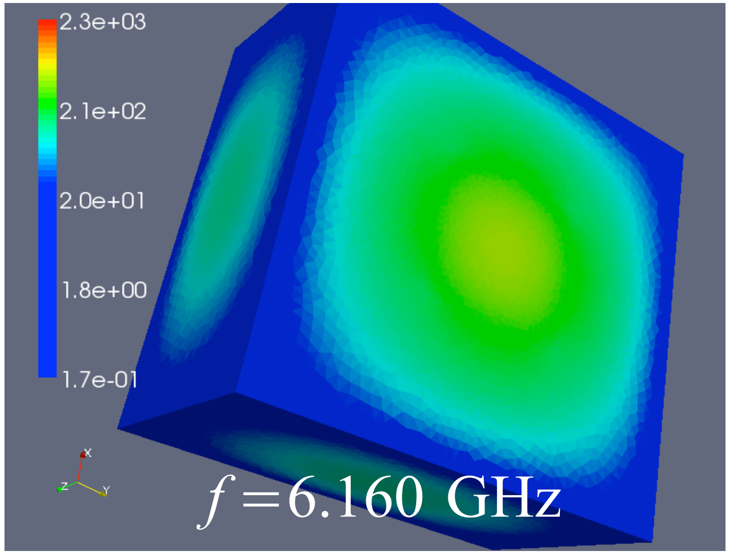
Quality factor: $Q = \frac{\mathrm{Re}(\lambda)}{2 \times \mathrm{Im}(\lambda)}$.

Table : Numerical results of the lossy dielectrix block cavity

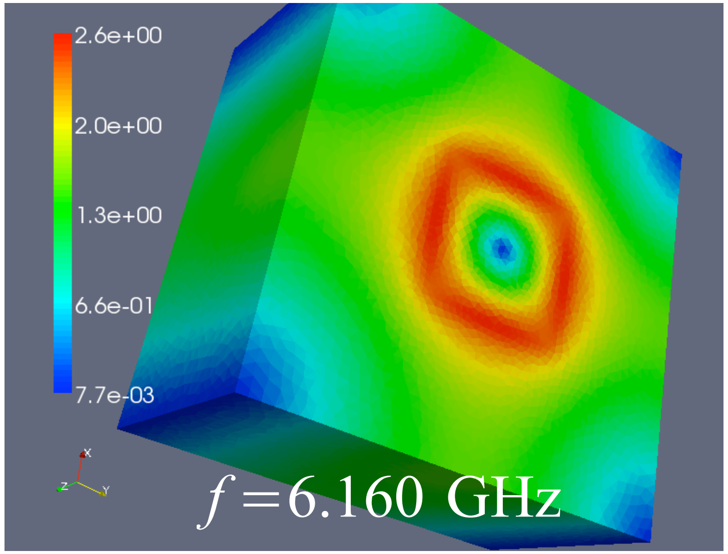
| mode | *f* (GHz) | Q |
|---|---|---|
| 1 | 6.171 | 11.2 |
| 2\|3 | 9.095 | 5.82 |
| 4\|5 | 11.41 | 7.48 |
| 6 | 11.42 | 5.49 |

Ref: S.J. Cooke and B. Levush. In 17th Particle Accelerator Conference, pages 2431–2433, May 1997.

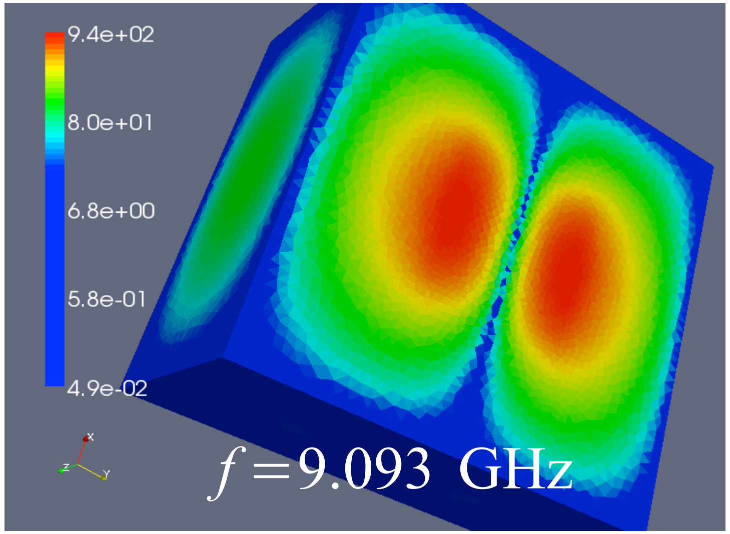
# Lossy Dielectric Block



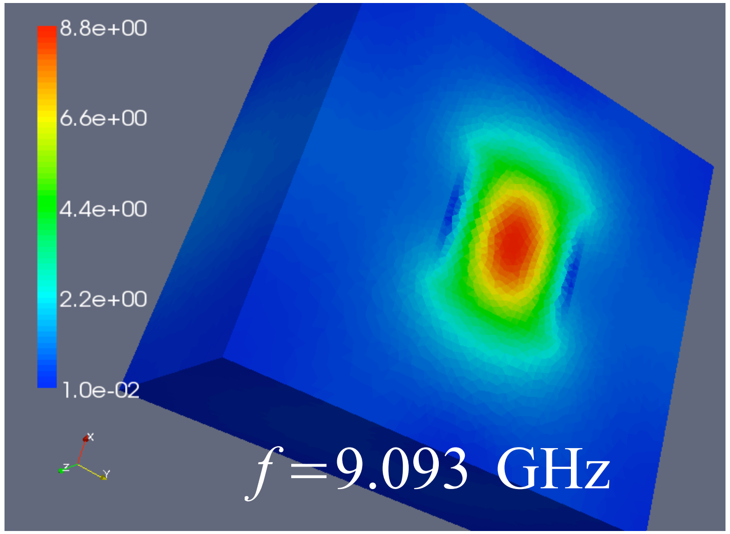

Mode 1: electric field




Mode 1: magnetic field




Mode 2: electric field




Mode 2: magnetic field

# Ohmically Lossy Material

- Considering conduction current density, then time-harmonic electric field vector wave equation is

$$\nabla\times(\mu^{-1}\nabla\times\boldsymbol{E}(\boldsymbol{x}))+i\,k_0\,\sigma\,Z_0\,\boldsymbol{E}-k_0^2\,\varepsilon_r\,\boldsymbol{E}(\boldsymbol{x})=\boldsymbol{0},\quad x\in\Omega\,.$$

$\sigma$ is ohmic conductivity.
$Z_0\approx 377\,\Omega$ is characteristic impedance in free space.

- FEM yields a constrained *quadratic* eigenproblem

$$T(\lambda)\,x:=A\,\boldsymbol{x}+\lambda\,R\,\boldsymbol{x}-\lambda^2 M\,\boldsymbol{x}=\boldsymbol{0},\qquad C^T\boldsymbol{x}=\boldsymbol{0}\,.$$

Entry of $R$: $r_{ij}=i\int_\Omega\sigma\,Z_0\,\boldsymbol{N}_i\cdot\boldsymbol{N}_j\,d\,\boldsymbol{x}\,.$

# Ohmically Lossy Material

- Linearize the quadratic eigenvalue problem

$$\begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{pmatrix} = \lambda \begin{bmatrix} -R & M \\ I & 0 \end{bmatrix} \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{pmatrix},$$

$$\begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix}^T \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{pmatrix} = \boldsymbol{0}.$$

Identity matrix

Then it is solved by JDQZ eigensolver.

- Dissipated energy now given as:

$$U_d = \frac{\varepsilon_0}{2} \int_\Omega \mathrm{Im}(\varepsilon_r) |\boldsymbol{E}|^2 d\boldsymbol{x} \longrightarrow U_d = \frac{1}{2} \int_\Omega \sigma |\boldsymbol{E}|^2 d\boldsymbol{x}$$

# Nonlinear Jacobi Davidson method (NLJD)

- Algorithm :

```
T(λ)x=0, choose start vector x_0;  it=0.
when it ≤ it_max do
    Solve projected nonlinear eigenproblem, V_k^T T(λ_k) V_k y = 0.
    Compute a Ritz pair closest to target.
    if the associated residual is small enough
        Return the Ritz pair.
    end if
    Find an approximate solution t of correction equation.
    Project t in null space of C^T.
    Orthonormalize t against previous search vectors.
    Expand search space with t.  it++.
end
```

Krylov solver

Divergence-free condition

Modified Gram-Schmidt

# AztecOO

- Correction equation in NLJD:

$$\left(I - \frac{p_k u_k^H}{u_k^H p_k}\right) T(\lambda_k) \left(I - \frac{u_k u_k^H}{u_k^H u_k}\right) t_k = -r_k, \quad t \perp u_k,$$

where $\lambda_k$ : approximation of eigenvalue, $u_k$ : Ritz vector, $p_k = T'(\lambda_k) u_k$

- Solve by AztecOO
  - Epetra_Operator to calculate projection
  - Ifpack preconditioner: $K \approx T(\tau)$, tau : target

$$\tilde{T} = \left(I - \frac{p_k u_k^H}{u_k^H p_k}\right) T(\tau) \left(I - \frac{u_k u_k^H}{u_k^H u_k}\right), \tilde{K} = \left(I - \frac{p_k u_k^H}{u_k^H p_k}\right) K \left(I - \frac{u_k u_k^H}{u_k^H u_k}\right),$$

⇨ $$\tilde{K}^{-1} \tilde{T} t_k = -\tilde{K}^{-1} r_k, \quad t \perp u_k,$$

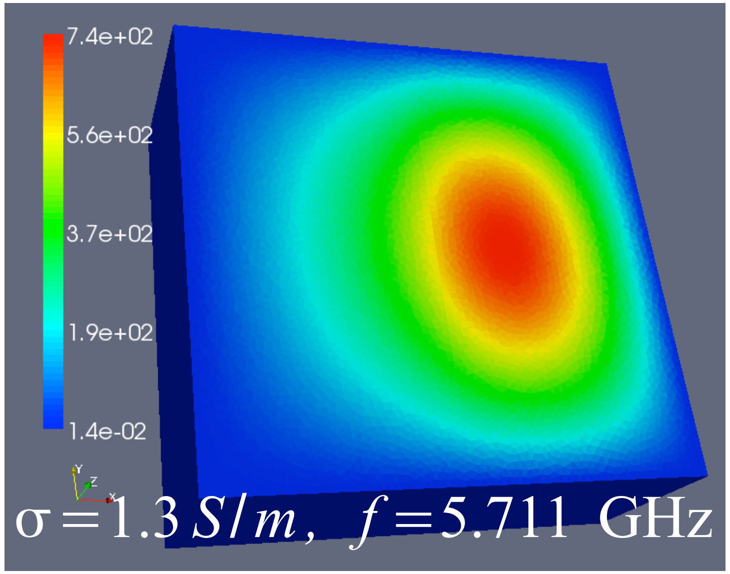
# Example of half-filled rectangle cavity

- **#Tetrahedra** : 306'001
- **Processors** : 32
- **Number of eigenvalues** : 1
- **Tolerance** : 1.0e-6(JDQZ), 1.0e-2(NLJD)
- **Target** : [100.0]
- **Krylov solvers** : Bi-CGStab, GMRES(m)
- **Preconditioners** : Diagonal, Block Jacobi-LU/ILU
- **Max iteration of correction equation** : 50
- **Jmin** : 5, **Jmax** : 10

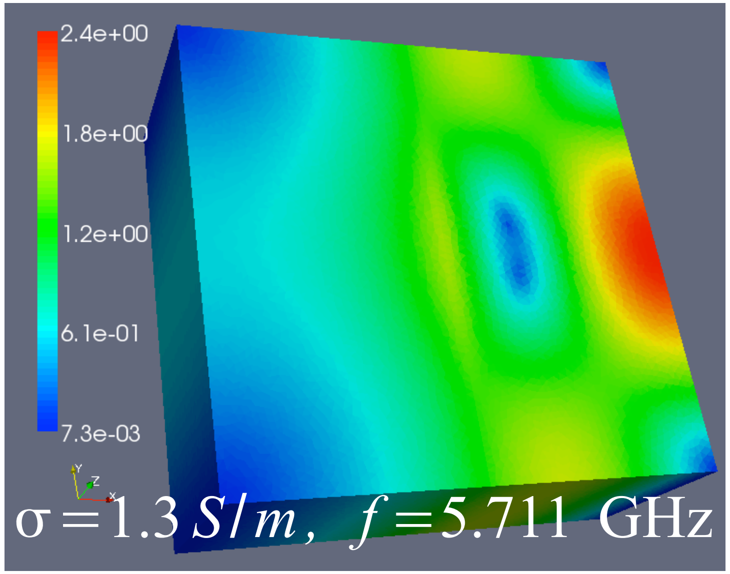
# Half-filled Rectangle Cavity

**Table**: dominant mode of a rectangle cavity $(22.86\times22.86\times10.16\,mm^3)$.
Half of the cavity is filled by conductor with $\varepsilon_r = 2.0$ and varying $\sigma$.
$\tilde{\omega} = \mathrm{Re}(\sqrt{\lambda})c + i\,\mathrm{Im}(\sqrt{\lambda})c$, $c$ : the speed of light

| $\sigma$ (S/m) | analytical solution for $\tilde{\omega}/(2\pi)$ | $\tilde{\omega}/(2\pi)$ computed by Femaxx |
|---|---|---|
| 0.1 | $7.379 + j0.354$ | $7.379 + j0.354$ |
| 0.5 | $7.236 + j1.819$ | $7.236 + j1.817$ |
| 1.0 | $6.579 + j3.864$ | $6.579 + j3.864$ |
| 1.3 | $5.711 + j5.197$ | $5.711 + j5.198$ |




electric field




magnetic field

# Half-filled Rectangle Cavity

- Linear element order
  - JDQZ eigensolver with linearization (dof:664'130)

| preconditioner | outer iteration | inner iteration | average | overall time | build time | solver time | "speed-up" |
|---|---|---|---|---|---|---|---|
| diag | 46 | 667 | 14.5 | 67.51 | 0.01 | 49.18 | |
| blkj-ilu | 34 | 348 | 10.24 | 64.88 | 1.17 | 35.9 | 1.04 |
| blkj-lu | 37 | 276 | 7.46 | 141.39 | 40.19 | 85.56 | 0.48 |

  - NLJD eigensolver, Bi-CGStab (dof:332'065)

| preconditioner | outer iteration | inner iteration | average | overall time | build time | solver time | "speed-up" |
|---|---|---|---|---|---|---|---|
| diag | 18 | 597 | 35.12 | 39.91 | 0.01 | 20.92 | |
| blkj-ilu | 17 | 366 | 22.88 | 34.72 | 1.27 | 16.18 | 1.15 |
| blkj-lu | 30 | 69 | 2.46 | 81.57 | 40.64 | 29.92 | 0.49 |

# Half-filled Rectangle Cavity

- Quadratic element order (dof:1'852'810)
  - NLJD, Bi-CGStab, 2-level hierarchical basis prec

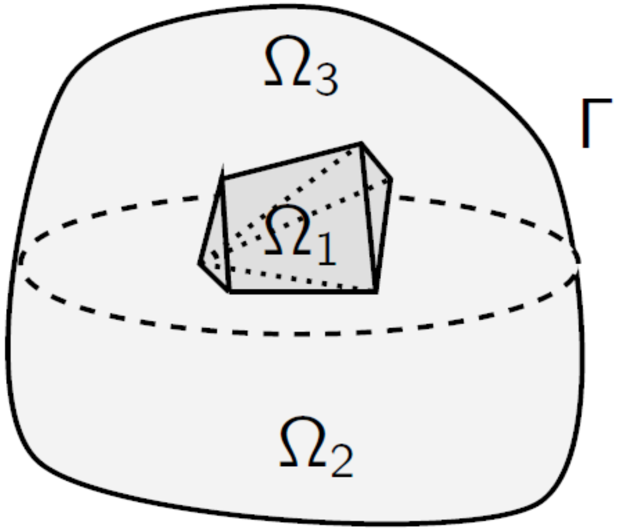
| preconditioner | outer iteration | inner iteration | average | overall time | build time | solver time | "speed-up" |
|---|---|---|---|---|---|---|---|
| diag | 53 | 1306 | 24.64 | 579.78 | 6.07 | 496.7 | |
| blkj-ilu | 29 | 366 | 12.62 | 261.00 | 7.3 | 177.74 | 2.22 |
| blkj-lu | 23 | 160 | 6.96 | 307.44 | 46.58 | 183.83 | 1.89 |

  - NLJD, GMRES(30), 2-level hierarchical basis prec

| preconditioner | outer iteration | inner iteration | average | overall time | build time | solver time | "speed-up" |
|---|---|---|---|---|---|---|---|
| diag | 44 | 852 | 21.85 | 384.91 | 0.05 | 259.91 | |
| blkj-ilu | 26 | 448 | 19.48 | 323.64 | 7.30 | 233.31 | 1.19 |
| blkj-lu | 19 | 202 | 11.88 | 360.77 | 46.30 | 232.04 | 1.07 |

# Microwave Antenna

- *Bounded* computational domain $\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_{3.}$




$\Omega_1$ : antenna device
$\Omega_2$ : substrate
$\Omega_3$ : environment
$\Gamma$ : absorbing boundary
$\Omega$ : arbitrary shape, usually sphere

- 1st order absorbing boundary condition (ABC):

$$\boldsymbol{n} \times \nabla \times \boldsymbol{E}(\boldsymbol{x}) = -i k_0 \sqrt{\mu_r \varepsilon_r}\, \boldsymbol{n} \times (\boldsymbol{n} \times \boldsymbol{E}(\boldsymbol{x})), \quad \boldsymbol{x} \in \Gamma, k_0 = \sqrt{\lambda}.$$

# Microwave Antenna

- FEM yields a constrained *quadratic* eigenproblem

$$A\boldsymbol{x} + \lambda R\boldsymbol{x} - \lambda^2 M\boldsymbol{x} = \boldsymbol{0}, \qquad C^T\boldsymbol{x} = \boldsymbol{0}.$$

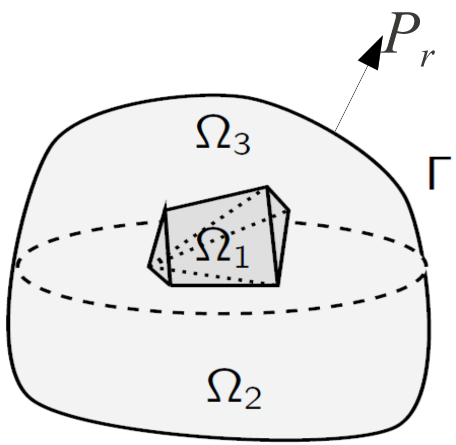
$$a_{ij} = \int_\Omega \mu_r^{-1} (\nabla \times \boldsymbol{N}_i) \cdot (\nabla \times \boldsymbol{N}_j)$$

$$r_{ij} = i \int_\Gamma \sqrt{\varepsilon_r / \mu_r} (\boldsymbol{n} \times \boldsymbol{N}_i) \cdot (\boldsymbol{n} \times \boldsymbol{N}_j)$$

$$m_{ij} = \int_\Omega \varepsilon_r \boldsymbol{N}_i \cdot \boldsymbol{N}_j$$

$$c_{il} = \int_\Omega \varepsilon_r \boldsymbol{N}_i \cdot \nabla N_l$$

integral on boundary



# Example of microwave antenna

- **#Tetrahedra** : 645'868
- **Processors** : 32
- **Number of eigenvalues** : 1
- **Tolerance** : 1.0e-3(JDQZ), 1.0e-3(NLJD)
- **Target** : [0.1]
- **Krylov solvers**: Bi-CGStab, GMRES(m)
- **Preconditioners** : Diagonal, Block Jacobi-LU/ILU
- **Max iteration of correction equation** : 50
- **Jmin** : 5, **Jmax** : 10

# Dielectric Resonant Antenna (DRA)

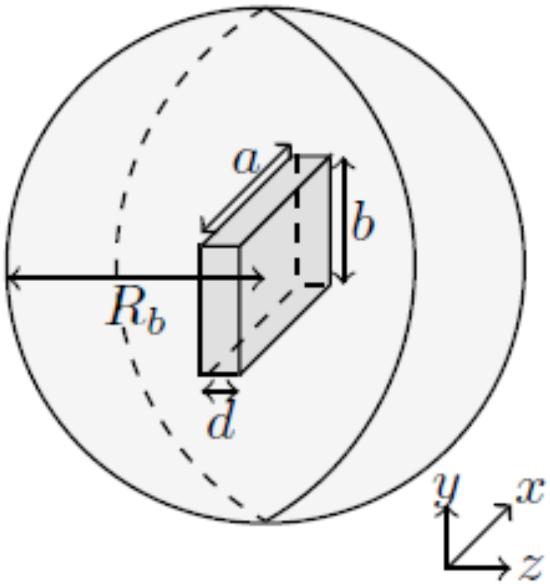
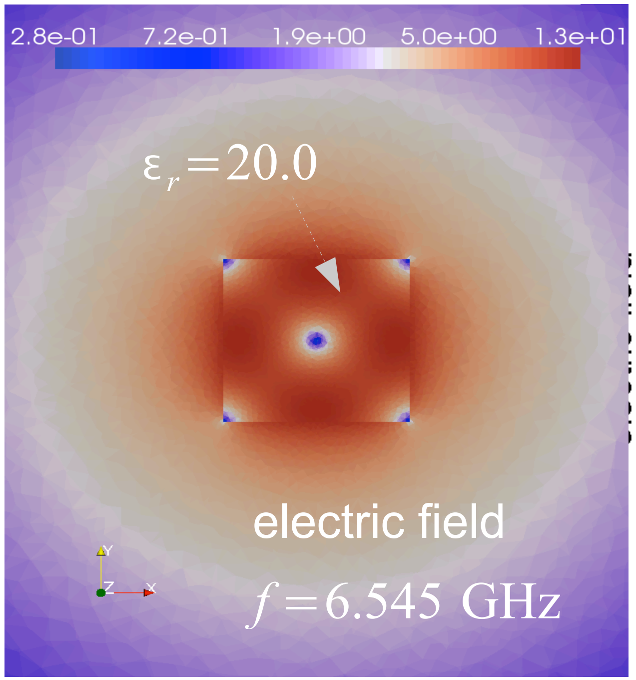
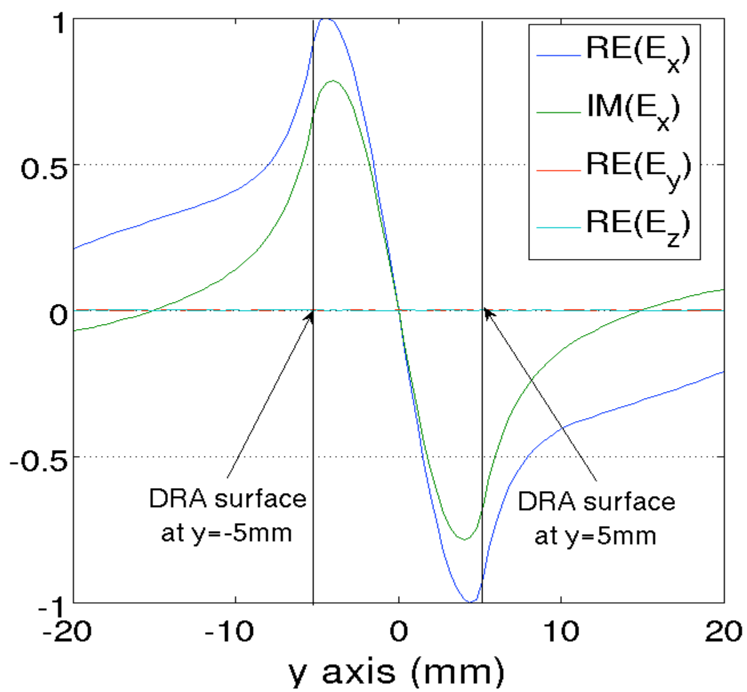



**Table**: Dominant mode of several DRA.

| $\varepsilon_r$ | a (mm) | b (mm) | d (mm) | $f$ (GHz) | $f_{\mathrm{MI}}$ (GHz) | Diff. (%) | $Q_1$ |
|---|---|---|---|---|---|---|---|
| 79.46 | 7.45 | 7.45 | 2.98 | 4.644 | 4.346 | 6.9 | 176.8 |
| 37.84 | 8.60 | 8.60 | 2.58 | 6.221 | 5.934 | 4.8 | 31.1 |
| 37.84 | 7.45 | 7.45 | 3.51 | 5.614 | 5.337 | 5.2 | 50.8 |
| 20.0 | 10 | 10 | 4 | 6.545 | 6.409 | 2.1 | 13.3 |

$f$, $Q_1$ and $Q_2$ are numerical results by using 1st order ABC;
$f_{MI}$, $Q_{MI}$ are theoretically determined results by using perfect magnetic conductor (PMC) boundary conditions.




**Remark:**
Electric field plotted over y-axis:
$E_y \approx 0$,
$E_x \approx A k_y \sin(k_y y)$.


Ref:
1, R. K. Mongia and A. Ittipiboon. IEEE Trans. Antennas Propag., 45(9):1348-1356 (1997).

2, H. Guo, B. Oswald, and P. Arbenz, Opt. Express 20, 5481-5500 (2012).

# Microwave Antenna

- Linear element order
  - JDQZ solver with linearization (dof:162'976)

| preconditioner | outer iteration | inner iteration | average | overall time | build time | solver time | “speed-up” |
|---|---|---|---|---|---|---|---|
| diag | 20 | 988 | 49.78 | 31.87 | 0 | 21.55 | |
| blkj-ilu | 18 | 881 | 48.94 | 19.03 | 0.43 | 8.93 | 1.67 |
| blkj-lu | 18 | 259 | 14.39 | 22.75 | 0.3 | 14.48 | 1.40 |

  - NLJD solver ,Bi-CGStab(dof:81'488)

| preconditioner | outer iteration | inner iteration | average | overall time | build time | solver time | “speed-up” |
|---|---|---|---|---|---|---|---|
| diag | 19 | 559 | 29.42 | 14.44 | 0 | 5.35 | |
| blkj-ilu | 19 | 587 | 30.89 | 14.60 | 0.4 | 5.99 | 0.99 |
| blkj-lu | 20 | 311 | 15.55 | 14.53 | 0.42 | 7.01 | 0.99 |

# Microwave Antenna

- Quadratic element order(dof:439'188)
  - NLJD solver, Bi-CGStab, 2-level hierarchical basis prec

| preconditioner | outer iteration | inner iteration | average | overall time | build time | solver time | “speed-up” |
|---|---|---|---|---|---|---|---|
| diag | 29 | 745 | 25.69 | 47.27 | 0.04 | 32.77 | |
| blkj-ilu | 17 | 325 | 19.12 | 31.11 | 0.62 | 14.31 | 1.52 |
| blkj-lu | 17 | 223 | 13.12 | 36.39 | 0.81 | 20.89 | 1.30 |

NLJD, GMRES(30), 2-level hierarchical basis prec

| preconditioner | outer iteration | inner iteration | average | overall time | build time | solver time | “speed-up” |
|---|---|---|---|---|---|---|---|
| diag | 35 | 952 | 30.71 | 77.46 | 0.02 | 48.78 | |
| blkj-ilu | 20 | 559 | 31.06 | 63.90 | 1.42 | 37.30 | 1.76 |
| blkj-lu | 33 | 1035 | 35.69 | 149.32 | 2.96 | 117.72 | 0.67 |

# Summary

- Femaxx: a parallel 3D finite element eigensolver package.
- Simulation tool for advanced electromagnetic structures.

| **eigensolver** | **problem type** | **applications** |
|---|---|---|
| JDSYM | Generalized real symmetric eigenvalue problem | lossless resonant cavities |
| JDQZ | Generalized non-Hermitian eigenvalue problem; Quadratic eigenvalue problem | dielectric lossy material; ohmically lossy material; plasmonic nanostructures |
| NLJD | Nonlinear eigenvalue problem | ohmically lossy material; plasmonic nanostructures |